

Theory of Weak Hypernuclear Decay*

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Abstract

The weak nonmesonic decay of Λ -hypernuclei is studied in the context of a one-meson-exchange model. Predictions are made for the decay rate, the p/n stimulation ratio and the asymmetry in polarized hypernuclear decay.

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1 Introduction

A great deal of attention has been focused over the last decade or so on the properties of Λ -hypernuclei, the study of which has yielded a rich store of valuable information concerning the $\Lambda - N$ interaction.[1] Typically such hypernuclei are produced via the (K^-, π^-) or (π^+, K^+) reaction, employing kinematics wherein the resulting Λ is produced with relatively low momentum. The Λ is generally not formed in the hypernuclear ground state, but rather proceeds there via emission of a series of γ -rays, the study of which yields the hypernuclear levels. However, the purpose of the present paper is not to study of this cascade process, but rather to analyze what happens once the Λ has finally reached its ground ($1s_{\frac{1}{2}}$) state. In fact, the Λ then decays weakly, and there are intriguing aspects of this process which form the topic of this paper.

The properties of the lambda hyperon are familiar. Having a mass of 1116 MeV, zero isospin and unit negative strangeness, it decays nearly 100% of the time via the nonleptonic mode $\Lambda \rightarrow N\pi$ and details can be found in the particle data tables[2]

$$\Gamma_{\Lambda} = \frac{1}{263 \text{ ps}} \quad \text{B.R. } \Lambda \rightarrow \begin{cases} p\pi^- & 64.1\% \\ n\pi^0 & 35.7\% \end{cases} \quad (1)$$

The decay can be completely described in terms of an effective Lagrangian with two phenomenological parameters

$$\mathcal{H}_w = g_w \bar{N}(1 + \kappa\gamma_5)\vec{\tau} \cdot \vec{\pi}\Lambda \quad (2)$$

where $g_w = 2.35 \times 10^{-7}$, $\kappa = -6.7$ and Λ is defined to occupy the lower entry of a two component column isospinor s — $\Lambda \equiv \Lambda s$ with $s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

However, it was realized early on by Primakoff and Cheston that when the Λ is bound in a hypernucleus, its decay properties are altered dramatically.[3] The kinematics for free $\Lambda \rightarrow N\pi$ decay at rest give the energy and momentum of the final state nucleon

$$T_N = \frac{(m_{\Lambda} - m_N)^2 - m_{\pi}^2}{2m_{\Lambda}} \approx 5 \text{ MeV}, \quad p_N = \sqrt{(T_N + m_N)^2 - m_N^2} \approx 100 \text{ MeV}/c \quad (3)$$

Thus, p_N is less than the nuclear Fermi momentum for all but the lightest nuclei, and the $N\pi$ decay of a Λ -hypernucleus is thus Pauli blocked. (Actually this suppression is even stronger than indicated above since typically the Λ is bound by 5-25 MeV.)

A simple estimate of this suppression is given within a simple shell model, wherein, taking free space kinematics (neglecting any effects of binding energy or of wavefunction distortion), and recognizing the fact that the pion recoils against the nucleus as a whole instead of a single nucleon, one finds the simple expression[4]

$$\frac{1}{\Gamma_\Lambda} \Gamma_{\Lambda \rightarrow N\pi} = 1 - \frac{1}{2} \sum_{nj\ell} N_{nj\ell} |\langle nj\ell | j_\ell(k_\pi r) | 1s_{\frac{1}{2}} \rangle|^2 \quad (4)$$

where $N_{nj\ell}$ is the occupation number for the indicated state and $k_\pi \sim 100$ MeV/c is the pion momentum. The result of this calculation reveals that the importance of such pionic decays rapidly falls as a function of nuclear mass, as shown in Figure 1.[7]

However, while the existence of the nuclear medium suppresses the $N\pi$ mode, it also opens up a completely new possibility, that of the nucleon-stimulated decay— $\Lambda N \rightarrow NN$ —which is, of course, unavailable to a free Λ . [5] This reaction is the $\Delta S = 1$ analog of the weak $NN \rightarrow NN$ reaction responsible for nuclear parity violation,[6] but with the difference that the weak parity-conserving $\Lambda N \rightarrow NN$ decay is *also* observable (in the $NN \rightarrow NN$ case the weak parity-conserving component is, of course, dwarfed by the strong interactions.) Note that the energy and momentum available in this process are, if shared equally by both outgoing nucleons,

$$T_N \approx \frac{1}{2}(m_\Lambda - m_N) \approx 90 \text{ MeV}, \quad p_N = \sqrt{(T_N + m_N)^2 - m_N^2} \approx 420 \text{ MeV}/c \quad (5)$$

which is well above a typical Fermi energies and momenta. Thus the Pauli effect does not significantly suppress the nonmesonic mode, and consequently the importance of the nonmesonic (NM) channel compared to its mesonic counterpart is expected to increase rapidly with A . This prediction is fully borne out experimentally, as shown in Figure 2. It is evident that once $A \geq 10$ the mesonic decay becomes but a small fraction of the dominant nonmesonic process.[7]

The dominant mode of hypernuclear weak decay then is not the pionic mode favored by a free Λ but becomes rather the far more complex $\Lambda N \rightarrow$

Figure 1: Ratio of calculated rate of $N\pi$ decay of hypernuclei to that of a free Λ as a function of nuclear mass number.

Figure 2: Measured ratio of nonmesonic ($\Lambda N \rightarrow NN$) to mesonic ($\Lambda \rightarrow N\pi$) hypernuclear decay as a function of nuclear mass number.

	${}^5_{\Lambda}\text{He}$	${}^{11}_{\Lambda}\text{B}$	${}^{12}_{\Lambda}\text{C}$
$\frac{1}{\Gamma_{\Lambda}}\Gamma_{NM}$	0.41 ± 0.14	$1.25 \pm 0.16^*$	1.14 ± 0.20
$\Gamma_{NM}(p/n)$	1.07 ± 0.58	$0.96^{+0.8}_{-0.4}$	$0.75^{+1.5}_{-0.35}$

Table 1: Experimental BNL data for nonmesonic hyperon decay. *Note that we have scaled the experimental number in order to remove the mesonic decay component.

NN process.[8] The observables which can be measured experimentally and should be confronted with theoretical predictions include

- i) the overall nonmesonic decay rate Γ_{NM} ;
- ii) the ratio of proton-stimulated ($\Lambda p \rightarrow np$) to neutron-stimulated ($\Lambda n \rightarrow nn$) decay— $\Gamma_{NM}^p/\Gamma_{NM}^n \equiv \Gamma_{NM}(p/n)$;
- iii) the ratio of parity-violating to parity-conserving decay— $\Gamma_{NM}^{PV}/\Gamma_{NM}^{PC} \equiv \Gamma_{NM}(PV/PC)$ —which is measured, *e.g.*, via the proton asymmetry in polarized hypernuclear decay;
- iv) final state n,p decay spectra;
- v) *etc.*

The present experimental situation is somewhat limited. Most of the early experiments in the field employed bubble chamber or emulsion techniques. It was therefore relatively straightforward to determine the ratio of the decay rates of the two modes, but much more difficult to measure the absolute rates. This changed when an early Berkeley measurement on ${}^{16}_{\Lambda}\text{O}$ yielded the value[9]

$$\frac{\Gamma({}^{16}_{\Lambda}\text{O})}{\Gamma_{\Lambda}} = 3 \pm 1 \quad (6)$$

However, this was still a very low statistics experiment with sizable background contamination. Recently a CMU-BNL-UNM-Houston-Texas-Vassar collaboration undertook a series of direct timing—fast counting—hypernuclear lifetime measurements yielding the results summarized in Table 1.[10]

Figure 3: Meson exchange picture of nuclear parity violation.

In addition, there exist a number of older emulsion measurements in light ($A \leq 5$) hypernuclei, details of which can be found in a recent review article.[8] The only experimental numbers for heavy systems are obtained from delayed fission measurements on hypernuclei produced in \bar{p} -nucleus collisions and are of limited statistical precision[11]

$$\tau_{\Lambda}^{(238\text{U})} = (1_{0.5}^{+1}) \times 10^{-10} \text{ sec.} \quad \tau_{\Lambda}^{(209\text{Bi})} = (2.5_{-1.0}^{+2.5}) \times 10^{-10} \text{ sec.} \quad (7)$$

The theoretical problem of dealing with a weak two-body interaction within the nucleus has been faced previously in the context of nuclear parity violation, and one can build on what has been learned therein.[12] Specifically, the weak interaction at the quark level is shortranged, involving W,Z-exchange. However, because of the hard core repulsion the NN effects are modelled in terms of long-range one-meson exchange interaction, just as in the case of the conventional strong nucleon-nucleon interaction,[13] but now with one vertex being weak and parity-violating while the second is strong and parity-conserving—*cf.* Figure 3. The exchanged mesons are the lightest ones— π^{\pm} , ρ , ω —associated with the longest range. (Exchange of neutral spinless mesons is forbidden by Barton’s theorem.[14])

A similar picture of hypernuclear decay can then be constructed, but with important differences. While the basic meson-exchange diagrams appear as before, the weak vertices must now include both parity-conserving *and* parity-violating components, and the list of exchanged mesons must be expanded

to include both neutral spinless objects (π^0, η^0) as well as strange mesons (K, K^*), as first pointed out by Adams.[15] Thus the problem is considerably more challenging than the corresponding and already difficult issue of nuclear parity violation.

One of the significant problems in such a calculation involves the evaluation of the various weak amplitudes. Indeed, the only weak couplings which are completely model-independent are those involving pion emission, which are given in Eqn. 2. In view of this, a number of calculations have included *only* this longest range component. Even in this simplified case, however, there is considerable model-dependence, as the results are strongly sensitive to the short-ranged correlation function assumed for the nucleon-nucleon interaction, as will be seen. Below we shall review previous theoretical work in this area and detail our own program, which involves a systematic quark model- (symmetry-) based evaluation of weak mesonic couplings to be used in hypernuclear decay calculations.

A brief outline of our paper is as follows. In section 2 we employ the quark model in order to construct the weak potentials which will be used to study the nonmesonic decay process. In sections 3,4 we apply these potentials in the regime of nuclear matter, finite nuclei respectively. Finally, we summarize our results in a concluding section 5.

2 Hypernuclear Decay: Effective Interaction

As discussed above, a primary challenge in the theoretical analysis of hypernuclear weak decay is in the calculation of the weak baryon-baryon-meson (BB'M) vertices, since the strong couplings are known reasonably well either from direct measurement or from the approximate validity of vector dominance. However, in the case of the weak interaction only $\Lambda N \pi$ vertices are accessible experimentally—evaluation of any remaining weak couplings requires a model. Our approach is based on the quark model together with a generalization of techniques previously developed to deal with nonleptonic processes.[16]

We begin with the parity-conserving weak interaction, which we evaluate using a pole model and the diagrams shown in Figure 4. What is required then is the evaluation of two-body matrix elements

$$\langle B' | \mathcal{H}_w^{(+)} | B \rangle, \langle P' | \mathcal{H}_w^{(+)} | P \rangle, \langle V' | \mathcal{H}_w^{(+)} | V \rangle \quad (8)$$

Figure 4: Pole model diagrams used in evaluation of the parity conserving component of $\Lambda N \rightarrow NN$.

where $\mathcal{H}_w^{(+)}$ is the parity-conserving $\Delta S = 1$ weak Hamiltonian and B,P,V represent baryons, pseudoscalar, vector mesons respectively. Although quark model based, the clearest way to characterize our results is in terms of the symmetry $SU(6)_w$ which is a generalization of $SU(6)$ allowing a relativistically correct theory for arbitrary boosts along one direction \hat{z} . [17] This is sufficient for a full treatment of weak nonleptonic vertices. The first applications to weak phenomenology were those of McKellar and Pick [18] and of Balachandran et al., [19] while a comprehensive $SU(6)_w$ treatment of nuclear parity violation was presented in ref. 10.

We begin by writing in $SU(3)$ tensor notation the $\Delta S = 1$ weak nonleptonic Hamiltonian

$$\mathcal{H}_w(\Delta S = 1) = \frac{G_v}{2\sqrt{2}} \cos \theta_c \sin \theta_c \{J_{\mu 1}^2, J_3^{\mu 1}\}_+ + \text{h.c.} \quad (9)$$

where $J_{\mu j}^i = (V + A)_{\mu j}^i$ is the weak hadronic current with $SU(3)$ indices i, j , $G_v \approx 10^{-5} m_p^{-2}$ is the conventional weak coupling constant, and θ_c is the Cabibbo angle. Now following the procedure of Balachandran et al. [19] we

express the vector and axial currents in terms of $SU(6)_w$ currents P,Q,R,S

$$\begin{aligned}
A_{+b}^a &= -R_{2b-1}^{2a} & A_{-b}^a &= -R_{2b}^{2a-1} \\
V_{+b}^a &= iQ_{2b-1}^{2a} & V_{-b}^a &= -iQ_{2b}^{2a-1} \\
V_{3b}^a &= \frac{1}{2}(P_{2b}^{2a} + P_{2b-1}^{2a-1}) & A_{0b}^a &= \frac{1}{2}(P_{2b}^{2a} - P_{2b-1}^{2a-1}) \\
V_{0b}^a &= \frac{1}{2}(S_{2b}^{2a} + S_{2b-1}^{2a-1}) & A_{3b}^a &= \frac{1}{2}(S_{2b}^{2a} - S_{2b-1}^{2a-1})
\end{aligned} \tag{10}$$

The A,V indices a,b are $SU(3)$ indices while on P,Q,R,S they represent $SU(6)_w$ indices. For the weak parity-conserving Hamiltonian we find

$$\mathcal{H}_w^{(+)}(\Delta S = 1) = \frac{G_v}{\sqrt{2}} \cos \theta_c \sin \theta_c^{(+)} \mathcal{O}_{43}^{56} \tag{11}$$

where

$$\begin{aligned}
^{(+)}\mathcal{O}_{CD}^{AB} &= U_{\{D,2\}}^{\{1,B\}} + U_{\{C,1\}}^{\{2,A\}} + U_{[D,2]}^{[1,B]} + U_{[C,1]}^{[2,A]} \\
&+ W_{\{D,2\}}^{\{2,A\}} + W_{\{C,1\}}^{\{1,B\}} + W_{[D,2]}^{[2,A]} + W_{[C,1]}^{[1,B]} + W_D^A + W_C^B
\end{aligned} \tag{12}$$

with

$$\begin{aligned}
U_{CD}^{AB} &= S_C^A S_D^B - P_C^A P_D^B \\
W_{CD}^{AB} &= -R_C^A R_D^B - Q_C^A Q_D^B
\end{aligned} \tag{13}$$

and $[,]$ and $\{, \}$ represent respective antisymmetrization and symmetrization of $SU(6)_w$ indices. Thus $\mathcal{H}_w^{(+)}(\Delta S = 1)$ transforms as $\{405\} + \{189\} + \{35\}$. We can then identify the various ways in which to couple this Hamiltonian to two baryons or mesons.

$$\begin{aligned}
\alpha : & \quad [M_{35} \times M_{35}]_{35} \\
\beta : & \quad [M_{35} \times M_{35}]_{189} \\
\gamma : & \quad [M_{35} \times M_{35}]_{405} \\
\delta : & \quad [\bar{B}_{56} \times B_{56}]_{35} \\
\epsilon : & \quad [\bar{B}_{56} \times B_{56}]_{405}
\end{aligned} \tag{14}$$

Specifically, we find

$$\langle K^0 | \mathcal{H}_w^{(+)} | \pi^0 \rangle = -\frac{1}{\sqrt{2}} \alpha$$

$$\begin{aligned}
\langle K^+ | \mathcal{H}_w^{(+)} | \pi^+ \rangle &= \alpha - \beta \\
\langle K^0 | \mathcal{H}_w^{(+)} | \eta^0 \rangle &= -\frac{1}{\sqrt{6}}\alpha \\
\langle K^{*0}(0) | \mathcal{H}_w^{(+)} | \omega^0(0) \rangle &= \langle K^{*0}(\uparrow) | \mathcal{H}_w^{(+)} | \omega^0(\uparrow) \rangle = -\frac{1}{\sqrt{6}}\alpha \\
\langle K^{*0}(0) | \mathcal{H}_w^{(+)} | \rho^0(0) \rangle &= \langle K^{*0}(\uparrow) | \mathcal{H}_w^{(+)} | \rho^0(\uparrow) \rangle = -\frac{1}{\sqrt{2}}\alpha \\
\langle K^{*+}(0) | \mathcal{H}_w^{(+)} | \rho^+(0) \rangle &= \alpha + \beta \\
\langle K^{*+}(\uparrow) | \mathcal{H}_w^{(+)} | \rho^+(\uparrow) \rangle &= \alpha + \gamma
\end{aligned} \tag{15}$$

In order to proceed we shall assume the validity of the $\Delta I = \frac{1}{2}$ rule for the $\Delta S = 1$ weak Hamiltonian. There has, of course, been a great deal of theoretical work attempting to identify the origin of this result.[20] The best indication at present is that in the case of hyperon decay, the suppression of $\Delta I = \frac{3}{2}$ effects is associated with the so-called Pati-Woo theorem,[21] which guarantees the vanishing of $\langle B' | \mathcal{H}_w^{(+)} | B \rangle$ between color-singlet baryons, while in the case of kaon transitions the validity of the $\Delta I = \frac{1}{2}$ rule appears to be associated with presently incalculable long distance contributions.[22] It is not our purpose here to enter into the debate concerning the dynamical origin of the $\Delta I = \frac{1}{2}$ rule, only to note its existence in other $\Delta S = 1$ nonleptonic weak processes and to *assume* its validity in the realm of nonmesonic hypernuclear decay as one of our inputs. (We shall return to this issue later.)

This may be done through the introduction of the iso-spurion $s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Then we can write

$$\langle K | \mathcal{H}_w^{(+)} | \pi \rangle = A_{K\pi} K^\dagger \vec{\tau} \cdot \vec{\pi} s \tag{16}$$

so that

$$\langle K^+ | \mathcal{H}_w^{(+)} | \pi^+ \rangle = \sqrt{2} A_{K\pi} = -\sqrt{2} \langle K^0 | \mathcal{H}_w^{(+)} | \pi^0 \rangle \tag{17}$$

Comparison with eqn. 15 yields

$$\alpha = \sqrt{2} A_{K\pi} \quad \beta = 0 \tag{18}$$

Similar considerations for the vector meson matrix element $\langle K^* | \mathcal{H}_w^{(+)} | \rho \rangle$ give the additional result

$$\gamma = 0 \tag{19}$$

Thus, all two-body weak mesonic amplitudes are determined in terms of the single parameter $A_{K\pi}$, which can in turn be related via PCAC to the experimental amplitude for $K \rightarrow \pi\pi$ decay.[23] The only subtlety is that it is essential to take into account the momentum dependence of the three-body matrix elements required by simultaneous consistency with current algebra limits and with the symmetry requirement that $\langle \pi\pi | \mathcal{H}_w^{(-)} | K \rangle$ must vanish in the SU(3) limit.[24] For the decay $K^0 \rightarrow \pi^0\pi^0$ this implies

$$A(K^0 \rightarrow \pi^0\pi^0) \propto 2k^2 - q_{01}^2 - q_{02}^2 \quad (20)$$

Then we find

$$\begin{aligned} \lim_{q_{02} \rightarrow 0} \langle \pi_{q_{01}}^0 \pi_{q_{02}}^0 | \mathcal{H}_w^{(-)} | K_k^0 \rangle &= \frac{-i}{F_\pi} \langle \pi_{q_{01}}^0 | [F_{\pi^0}^5, \mathcal{H}_w^{(-)}] | K_k^0 \rangle \\ &= -\frac{i}{2F_\pi} \langle \pi_{q_{01}}^0 | \mathcal{H}_w^{(+)} | K_k^0 \rangle = \frac{k \cdot q_{01}}{2(m_K^2 - m_\pi^2)} \langle \pi^0 \pi^0 | \mathcal{H}_w^{(-)} | K^0 \rangle_{\text{expt.}} \end{aligned} \quad (21)$$

so that

$$A_{K\pi} \simeq -iF_\pi \frac{k \cdot q_0}{m_K^2 - m_\pi^2} \langle \pi^0 \pi^0 | \mathcal{H}_w^{(-)} | K^0 \rangle_{\text{expt.}} \equiv \tilde{A}_{K\pi} k \cdot q_0 \quad (22)$$

Having determined the two-body parity-conserving meson amplitudes, we turn to the corresponding baryon matrix elements. Here the Pati-Woo theorem guarantees the validity of the $\Delta I = \frac{1}{2}$ rule, and we can characterize the matrix elements in terms of two independent parameters $A_{N\Lambda}, A_{N\Sigma}$ defined via

$$\langle N | \mathcal{H}_w^{(+)} | \Lambda \rangle = A_{N\Lambda} \bar{N} \Lambda, \quad \langle N | \mathcal{H}_w^{(+)} | \Sigma \rangle = A_{N\Sigma} \bar{N} \vec{\tau} \cdot \vec{\Sigma} s \quad (23)$$

These quantities can be determined via current algebra/PCAC as before

$$\begin{aligned} \lim_{q_0 \rightarrow 0} \langle \pi^0 n | \mathcal{H}_w^{(-)} | \Lambda \rangle &= \frac{-i}{F_\pi} \langle n | [F_{\pi^0}^5, \mathcal{H}_w^{(-)}] | \Lambda \rangle = \frac{i}{2F_\pi} \langle n | \mathcal{H}_w^{(+)} | \Lambda \rangle \\ \lim_{q_0 \rightarrow 0} \langle \pi^0 p | \mathcal{H}_w^{(-)} | \Sigma^+ \rangle &= \frac{-i}{F_\pi} \langle p | [F_{\pi^0}^5, \mathcal{H}_w^{(-)}] | \Sigma^+ \rangle = \frac{i}{2F_\pi} \langle p | \mathcal{H}_w^{(+)} | \Sigma^+ \rangle \end{aligned} \quad (24)$$

Then assuming no momentum dependence for the baryon S-wave decay amplitude[2]

$$\begin{aligned} A_{N\Lambda} &\simeq -iF_\pi \langle \pi^0 n | \mathcal{H}_w^{(-)} | \Lambda \rangle = -i4.46 \times 10^{-5} \text{ MeV} \\ A_{N\Sigma} &\simeq -i\sqrt{2}F_\pi \langle \pi^0 p | \mathcal{H}_w^{(-)} | \Sigma^+ \rangle = i4.36 \times 10^{-5} \text{ MeV.} \end{aligned} \quad (25)$$

Figure 5: Baryon-baryon matrix element of the weak Hamiltonian.

Now although the specific technique used in order to obtain eqn. 15 was the symmetry $SU(6)_w$, it is straightforward to demonstrate that identical results are obtained in the simple valence quark model. Indeed since the $SU(6)_w$ indices $A = 1, 2 \dots 6$ correspond to quark flavor spin states— $1 = u \uparrow, 2 = u \downarrow, \dots 6 = s \downarrow$ —then the $SU(6)$ 405 tensors U_{CD}^{AB} contain four external indices which can be expressed by their action on four quark fields. In particular the combination of tensors which appears in the Hamiltonian is the same as the nonrelativistic reduction of

$$H_U = V_0 V_0 + A_0 A_0 - V_3 V_3 - A_3 A_3 \quad (26)$$

The W portion follows likewise from the transverse currents. In forming tensors from the baryon and meson fields with the same transformation properties as a given tensor U_{CD}^{AB} we must contract the $SU(6)_w$ indices that are not set equal to A, B, C, D . Thus the meson to meson amplitudes require no contraction, but the baryon to baryon vertices involve a summation

$$[\bar{B}_{56} \times B_{56}]_{405} \sim \bar{B}^{eab} B_{ecd} \quad (27)$$

Of course, several terms of this form must be symmetrized in order to form the required Hamiltonian. These observations can be combined to give a pictorial way in which to represent the group theoretical structure—instead of using $SU(6)_w$ we can utilize quark indices throughout. Then the parameters $a, b, \dots e$ correspond to the quark spin sums depicted in Figure 5.

Baryons and mesons are described by $SU(6)$ wavefunctions (which are equivalent to the tensors B_{abc} and M_{ab}) and the wavy line describes the

$SU(6)_w$ Hamiltonian. A study of indices in Eqn. 14 and Figure 4 reveals that the two approaches are equivalent. At this stage then the diagrams could be considered merely as simple ways by which to obtain the $SU(6)_w$ transformation properties. However, in a quark model they also have a well-defined dynamical meaning and can, in some approximation, be calculated. It is not our purpose here to undertake such a calculation, however. Indeed naive quark-only diagrams, which omit the all-important gluon and/or quark sea content cannot possibly convey the intricate dynamical details which characterize processes involving low energy strong interactions. An archetypical example is the empirical validity of the $\Delta I = \frac{1}{2}$ rule in $\Delta S = 1$ nonleptonic processes such as $K \rightarrow 2\pi, 3\pi$ and $B \rightarrow B'\pi$. Simple quark diagrams would suggest that the strength of the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ components of the weak interaction should be comparable. Inclusion of leading-log gluonic corrections via renormalization group methods provides an enhancement of the $\Delta I = \frac{1}{2}$ piece over its $\Delta I = \frac{3}{2}$ counterpart by a factor of 3-4, and the remaining factor of 5-6 required in order to agree with experiment arises (presumably) from subtle details of the of the strongly interacting particles involved. Calculations of such detail are well beyond the ability of present theoretical methods and consequently we shall simply utilize the semi-empirical results quoted in Eqns. 22, 25, which encode the details of strong and weak interaction dynamics in a set of experimental constants.

We can now, in terms of these results, construct any of the needed amplitudes— $T_w^{\Lambda N\pi}, T_w^{NNK}, T_w^{\Lambda N\rho}$, *etc.*—via the pole diagrams indicated in Figure 3. As a check of the validity of this technique, we evaluate the only amplitude which is experimentally accessible—the P-wave $\Lambda N\pi$ decay amplitude. We predict

$$T_w^{\Lambda N\pi} = \bar{N}\vec{\tau} \cdot \vec{\pi}\Lambda \left[Q_{\Lambda\Sigma\pi} \frac{1}{m_N - m_\Sigma} A_{N\Sigma} + Q_{NN\pi} \frac{1}{m_\Lambda - m_N} A_{N\Lambda} + Q_{\Lambda NK} \frac{1}{m_K^2 - m_\pi^2} \tilde{A}_{K\pi} m_\pi^2 \right] \quad (28)$$

where the strong couplings $Q_{BB'M}$ are defined via use of the generalized Goldberger-Treiman relation and PCAC/ $SU(3)$. [25]

$$Q_{B^i B^j M^a} = \frac{m_{B^i} + m_{B^j}}{\sqrt{2}F_\pi} [dd_{ija} + if f_{ija}] \quad (29)$$

and[26]

$$f + d = 1.26, \quad d/(f + d) \simeq 0.63 \quad (30)$$

from fits to semileptonic hyperon decay. Using

$$\begin{aligned} Q_{\Lambda\Sigma\pi} &= \frac{1}{\sqrt{3}}d \frac{m_\Lambda + m_N}{F_\pi} \\ Q_{NN\pi} &= -(f + d) \frac{m_N}{F_\pi} \\ Q_{\Lambda NK} &= -\frac{1}{2\sqrt{3}}(3f + d) \frac{m_\Lambda + m_N}{F_\pi} \end{aligned} \quad (31)$$

we find

$$T_w^{\Lambda n\pi^0}(\text{theo.}) = 1.35 \times 10^{-6} \quad (32)$$

to be compared with the experimental value[2]

$$T_w^{\Lambda n\pi^0}(\text{expt.}) = 1.61 \times 10^{-6} \quad (33)$$

Actually this near agreement should be considered somewhat fortuitous. Indeed it is well-known that use of $\langle B' | \mathcal{H}_w^{(+)} | B \rangle$ matrix elements as determined from current algebra/PCAC and an SU(3) fit to S-wave hyperon decay amplitudes— $F = -2.4D = -0.92 \times 10^{-7}$ —yields generally a rather poor fit to corresponding P-wave hyperon decay amplitudes, with values in some cases about 50% too small.[27] However, a slight shift in these parameters— $F = -1.8D = -1.44 \times 10^{-7}$ —yields a much better fit and reproduces nearly all amplitudes to within 15%.[16] Since for our purposes we require only the vertices $A_{N\Lambda}, A_{N\Sigma}$ and since these differ only by about 15% in the two parameterizations quoted above, we shall not worry about this difference, given the preliminary nature of our investigation. Indeed we have verified that calculations performed with either set of couplings are quite similar.

Having thus obtained a handle on the parity-conserving, or P-wave amplitudes, we move on to consider the parity-violating meson vertices, which are to be inserted into the diagrams shown in Figure 6 in order to generate two-body $\Lambda N \rightarrow NN$ operators.

Phenomenologically, assuming again only the $\Delta I = \frac{1}{2}$ rule, we can write $\mathcal{H}_w^{(-)}$ in terms of eight unknown couplings

$$\mathcal{H}_w^{(-)} = A_{\pi\Lambda N} \bar{N} \vec{\tau} \cdot \vec{\pi} \Lambda + A_{\eta\Lambda N} \bar{N} \eta \Lambda$$

Figure 6: Two body operators for parity-violating nonmesonic hypernuclear decay.

$$\begin{aligned}
& + C_{KNN} \bar{N} s K^\dagger N + D_{KNN} \bar{N} N K^\dagger s \\
& + A_{\rho\Lambda N} \bar{N} \gamma_\mu \gamma_5 \vec{\tau} \cdot \vec{\rho}^\mu \Lambda + A_{\omega\Lambda N} \bar{N} \gamma_\mu \gamma_5 \omega^\mu \Lambda \\
& + C_{K^*NN} \bar{N} s \gamma_\mu \gamma_5 K^{*\mu\dagger} N + D_{K^*NN} \bar{N} \gamma_\mu \gamma_5 N K^{*\mu\dagger} s
\end{aligned} \tag{34}$$

These various couplings may be interrelated via the $SU(6)_w$ symmetry scheme, or equivalently by use of the quark model. We find, in $SU(6)_w$, the Hamiltonian

$$\mathcal{H}_w^{(-)}(\Delta S = 1) = \frac{G_v}{\sqrt{2}} \cos \theta_c \sin \theta_c^{(-)} \mathcal{O}_{43}^{56} \tag{35}$$

where

$$\begin{aligned}
{}^{(-)}\mathcal{O}_{CD}^{AB} &= T_{\{C,1\}}^{[2,A]} + T_{[C,1]}^{\{2,A\}} - T_{\{D,2\}}^{[1,B]} - T_{[D,2]}^{\{1,B\}} \\
&+ V_{\{D,2\}}^{[2,A]} + V_{[D,2]}^{\{2,A\}} - V_{\{C,1\}}^{[1,B]} - V_{[C,1]}^{\{1,B\}} + V_C^B - V_D^A
\end{aligned} \tag{36}$$

with

$$\begin{aligned}
T_{CD}^{AB} &= P_C^A S_D^B - S_C^A P_D^B \\
V_{CD}^{AB} &= -i R_C^A Q_D^B + i Q_C^A R_D^B
\end{aligned} \tag{37}$$

Note that T describes the effects of currents along the boost direction (0,3), while V includes currents orthogonal to this direction (1,2 or +,-). The weak

Hamiltonian is seen to involve the representations

$$\mathcal{H}_w^{(-)} = 280 + \overline{280} + 35. \quad (38)$$

We can then define, in an obvious notation, the various ways in which to couple the two baryons and meson together in a CP-invariant fashion

$$\begin{aligned} a_T, a_V : & \quad [(\bar{B}B)_{35} \times M_{35}]_{280, \overline{280}} \\ b_T, b_V : & \quad [(\bar{B}B)_{405} \times M_{35}]_{280, \overline{280}} \\ c_V : & \quad [(\bar{B}B)_{35} \times M_{35}]_{35} \end{aligned} \quad (39)$$

In terms of these couplings we have (temporarily, for simplicity, omitting a_V, a_T contributions)

$$\begin{aligned} A_{\pi\Lambda N}^{b,c} &= -\frac{1}{\sqrt{6}}\left(\frac{1}{6}b_V - \frac{1}{12}b_T - \frac{1}{2}c_V\right) \\ A_{\rho\Lambda N}^{b,c} &= \frac{1}{\sqrt{6}}\left(-\frac{1}{6}b_V + \frac{1}{4}b_T + \frac{1}{2}c_V\right) \\ A_{\omega\Lambda N}^{b,c} &= \frac{1}{36\sqrt{2}}(4b_V - 5b_T - 6c_V) \\ C_{KNN}^{b,c} &= -\frac{1}{\sqrt{2}}\left(\frac{1}{36}b_V + \frac{1}{36}b_T - \frac{1}{3}c_V\right) \\ D_{KNN}^{b,c} &= \sqrt{2}\left(-\frac{5}{36}b_V + \frac{1}{9}b_T + \frac{1}{6}c_V\right) \\ C_{K^*NN}^{b,c} &= \frac{1}{9\sqrt{2}}(2b_V - b_T - 5c_V) \\ D_{K^*NN}^{b,c} &= \frac{1}{9\sqrt{2}}\left(-\frac{1}{2}b_V + b_T + c_V\right) \\ A_{\eta\Lambda N}^{b,c} &= \sqrt{3}A_{\pi\Lambda N} \end{aligned} \quad (40)$$

where for consistency with Lorentz invariance it is necessary to require $b_V = -b_T$. [12] Then $b_V (= -b_T)$ and c_V are determined from a fit to parity-violating hyperon decay

$$b_V = -b_T = -1.28c_V = -4.0 \times 10^{-7} \quad (41)$$

We now return to the couplings a_V, a_T . It has been shown in ref. 16 that such a -type coupling constants correspond to the so-called “factorization” terms [28] wherein the meson is coupled to the vacuum by one of the weak

currents in all possible ways consistent with Fierz reordering.[29] Such a term is very small in the case of pion emission because of the near conservation of the axial current. However, the factorization term is substantial in the case of vector meson vertices and, in fact, originally was the only term considered to contribute to vector meson emission. A naive evaluation of such terms explicitly violates the $\Delta I = \frac{1}{2}$ rule, as factorization reflects only the basic symmetry of the Hamiltonian. These couplings have been determined in ref. 16 as

$$a_T = \frac{1}{3}a_V = \frac{3}{5}G_v \cos \theta_c \sin \theta_c \langle \rho | V_\mu^3 | 0 \rangle \langle p | A^{3\mu} | p \rangle. \quad (42)$$

Since $\Delta I = \frac{3}{2}$ effects are suppressed with respect to their $\Delta I = \frac{1}{2}$ counterparts by a factor of twenty or so, we shall neglect them in this preliminary analysis of hypernuclear decay.¹ We then find the factorization contributions to the various couplings

$$\begin{aligned} A_{\pi\Lambda N}^a &= C_{KNN}^a = D_{KNN}^a = A_{\eta\Lambda N}^a = 0 \\ A_{\rho\Lambda N}^a &= \sqrt{3}a_T \\ A_{\omega\Lambda N}^a &= -\frac{1}{3}a_T \\ C_{K^*NN}^a &= \frac{30}{9}a_T \\ D_{K^*NN}^a &= \frac{8}{9}a_T. \end{aligned} \quad (43)$$

Having determined values for each of the weak couplings we can insert them into the meson exchange diagrams indicated in Figure 6 in order to determine the various weak $\Lambda N \rightarrow NN$ “potential” forms. Of course, we need also to know the strong interaction $BB'M$ vertices, for which we utilize

$$\begin{aligned} \mathcal{H}_{st} &= iQ_{\pi NN} \bar{N} \gamma_5 \vec{\tau} \cdot \vec{\pi} N + iQ_{\pi \Sigma \Lambda} \bar{\Sigma} \cdot \vec{\pi} \gamma_5 \Lambda \\ &+ iQ_{\eta NN} \bar{N} \gamma_5 \eta N + iQ_{\eta \Lambda \Lambda} \bar{\Lambda} \gamma_5 \eta \Lambda \\ &+ iQ_{KN\Lambda} \bar{N} \gamma_5 K \Lambda + iQ_{K\Sigma N} K^\dagger \vec{\Sigma} \cdot \vec{\tau} \gamma_5 N \end{aligned}$$

¹As argued above, this suppression does not follow in the simple quark model but rather requires consistent treatment of soft gluon effects. Thus, for example, recent “direct” quark calculations[30, 31] predict a substantial $\Delta I = \frac{3}{2}$ components but are open to question since they omit these important strong interaction effects. Thus in our approach we instead employ the empirical results and explicitly omit $\Delta I = \frac{3}{2}$ terms.

$$\begin{aligned}
& + iQ_{\rho NN}\bar{N}(\gamma_\mu - i\chi_V\frac{1}{2M}\sigma_{\mu\nu}q^\nu)\bar{\rho}^\mu \cdot \vec{\tau}N \\
& + iQ_{\rho\Sigma\Lambda}\vec{\bar{\Sigma}} \cdot \vec{\rho}^\mu(\gamma_\mu - i\chi_Y\frac{1}{2M}\sigma_{\mu\nu}q^\nu)\Lambda \\
& + iQ_{\omega NN}\bar{N}(\gamma_\mu - i\chi_Z\frac{1}{2M}\sigma_{\mu\nu}q^\nu)\omega^\mu N \\
& + iQ_{\omega\Lambda\Lambda}\bar{\Lambda}(\gamma_\mu - i\chi_L\frac{1}{2M}\sigma_{\mu\nu}q^\nu)\omega^\mu \Lambda \\
& + iQ_{K^*N\Lambda}\bar{N}(\gamma_\mu - i\chi_T\frac{1}{2M}\sigma_{\mu\nu}q^\nu)K^{*\mu}\Lambda \\
& + iQ_{K^*\Sigma N}K^{*\mu\dagger}\vec{\bar{\Sigma}} \cdot \vec{\tau}(\gamma_\mu - i\chi_G\frac{1}{2M}\sigma_{\mu\nu}q^\nu)N
\end{aligned} \tag{44}$$

where M is the nucleon mass and $q = p_i - p_f$ is the recoil momentum carried off by the meson. The strong pseudoscalar couplings are determined in terms of SU(3) symmetry and the generalized Goldberger-Treiman relation, as discussed earlier, yielding

$$\begin{aligned}
Q_{\pi NN} &= \frac{m_N}{F_\pi}(d + f) & Q_{\pi\Sigma\Lambda} &= \frac{m_\Sigma + m_\Lambda}{\sqrt{3}F_\pi}d \\
Q_{\eta NN} &= \frac{m_N}{\sqrt{3}F_\pi}(3f - d) & Q_{\eta\Lambda\Lambda} &= -\frac{2m_\Lambda}{\sqrt{3}F_\pi}d \\
Q_{KN\Lambda} &= -\frac{m_\Lambda + m_N}{2\sqrt{3}F_\pi}(d + 3f) & Q_{K\Sigma N} &= \frac{m_\Sigma + m_N}{2F_\pi}(d - f)
\end{aligned} \tag{45}$$

with f, d given in Eqn. 30.

The strong vector meson couplings can be found via the combined assumptions of SU(3) symmetry and vector dominance, whereby

$$\langle B'|V_\mu^i|B\rangle = \frac{m_\rho F_\pi}{m_i^2}\langle\phi_\mu^i B'|B\rangle \tag{46}$$

Putting this together with the nucleon-nucleon matrix element of the electromagnetic current

$$\langle N|V_\mu^{em}|N\rangle = \bar{N}[\gamma_\mu\frac{1}{2}(1 + \tau_3) - \frac{i}{2M}\sigma_{\mu\nu}q^\nu(1.85\tau_3 - 0.06)]N \tag{47}$$

we have

$$\begin{aligned}
\langle\phi^j B'|B\rangle &= \frac{-im_j^2}{m_\rho F_\pi}\epsilon^\mu \bar{B}'[-if_{jB'B}\gamma_\mu \\
&- i(-0.83if_{jB'B} + 2.87d_{jB'B})\sigma_{\mu\nu}q^\nu\frac{1}{2M}]B
\end{aligned} \tag{48}$$

Finally, we can evaluate the meson exchange diagrams contributing to the decay process, yielding for the parity-violating case

$$\begin{aligned}
V^{(-)}(r) = & \frac{i}{4}Q_{\pi NN}A_{\pi\Lambda N}(U - 3Z)[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)] \cdot \vec{f}_\pi^{(-)}(r) \\
& + \frac{i}{4}Q_{\eta NN}A_{\eta\Lambda N}[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)] \cdot \vec{f}_\eta^{(-)}(r) \\
& - \frac{i}{4}Q_{KNN}[(C_{KNN} + D_{KNN})U - (C_{KNN} - D_{KNN})Z] \\
& \times [T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)] \cdot \vec{f}_K^{(-)}(r) \\
& + \frac{i}{2}Q_{\rho NN}A_{\rho\Lambda N}(U - 3Z)\{[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T + (\vec{\sigma}_\Lambda - \vec{\sigma}_N)] \cdot \vec{f}_\rho^{(+)}(r) \\
& + (1 + \chi_V)[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)S - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)T] \cdot \vec{f}_\rho^{(-)}(r)\} \\
& + \frac{i}{2}Q_{\omega NN}A_{\omega\Lambda N}\{[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T + (\vec{\sigma}_\Lambda - \vec{\sigma}_N)] \cdot \vec{f}_\omega^{(+)}(r) \\
& + (1 + \chi_Z)[T(\vec{\sigma}_\Lambda - \vec{\sigma}_N)S - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)T] \cdot \vec{f}_\omega^{(-)}(r)\} \\
& + \frac{i}{2}Q_{K^*NN}[(C_{K^*NN} + D_{K^*NN})U - (C_{K^*NN} - D_{K^*NN})Z] \\
& \times \{[T(\vec{\sigma}_\Lambda + \vec{\sigma}_N)T - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)T] \cdot \vec{f}_{K^*}^{(+)}(r) \\
& + (1 + \chi_T)[T\vec{\sigma}_\Lambda - \vec{\sigma}_N)S - (\vec{\sigma}_\Lambda - \vec{\sigma}_N)T] \cdot \vec{f}_{K^*}^{(-)}(r)\} \tag{49}
\end{aligned}$$

where T,S are the triplet, singlet spin projection operators

$$T = \frac{1}{4}(3 + \vec{\sigma}_N \cdot \vec{\sigma}_\Lambda), \quad S = \frac{1}{4}(1 - \vec{\sigma}_N \cdot \vec{\sigma}_\Lambda) \tag{50}$$

while U,Z are the triplet, singlet isospin projection operators

$$U = \frac{1}{4}(3 + \vec{\tau}_N \cdot \vec{\tau}_\Lambda), \quad Z = \frac{1}{4}(1 - \vec{\tau}_N \cdot \vec{\tau}_\Lambda) \tag{51}$$

The radial dependence is given by

$$\begin{aligned}
\vec{f}_M^{(-)}(r) &= \left[\frac{i\vec{\nabla}_1 - i\vec{\nabla}_2}{2M}, f_M(r) \right] \\
\vec{f}_M^{(+)}(r) &= \left\{ \frac{i\vec{\nabla}_1 - i\vec{\nabla}_2}{2M}, f_M(r) \right\} \tag{52}
\end{aligned}$$

Figure 7: K-exchange diagram contributing to parity-conserving $\Lambda N \rightarrow NN$.

where $f_M(r) = \exp(-m_M r)/4\pi r$.

The parity-conserving potential is rather more complicated. Consider, *e.g.*, Figure 7 for which

$$V_K^{(+)}(r) = \frac{Q_{KN\Lambda}Q_{KN\Lambda}A_{\Lambda N}}{m_N - m_\Lambda}(U - Z)\{\}_K \quad (53)$$

where

$$\begin{aligned} \{\}_K &= -\frac{1}{12}(T - 3S)f_K^{[jj]}(r) - \frac{1}{4}(\sigma_\Lambda^j \sigma_N^k - \frac{1}{3}\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N \delta^{jk})f_K^{[jk]}(r) \\ \text{and} \\ f_M^{[jk]}(r) &= \left[\frac{i\nabla_1^j - i\nabla_2^j}{2M}, \left[\frac{i\nabla_1^k - i\nabla_2^k}{2M}, f_M(r) \right] \right]. \end{aligned} \quad (54)$$

To this must be added the appropriate diagrams for π, η exchange yielding

$$\begin{aligned} V_{P-tot}^{(+)}(r) &= V_K^{(+)}(r) + \frac{Q_{KN\Lambda}Q_{KN\Sigma}A_{\Sigma N}}{m_N - m_\Sigma}(U - Z)\{\}_K \\ &+ \frac{Q_{\pi\Sigma\Lambda}Q_{\pi NN}A_{\Sigma N}}{m_N - m_\Sigma}(U - 3Z)\{\}_\pi \\ &+ \frac{Q_{\pi NN}Q_{\pi NN}A_{\Lambda N}}{m_\Lambda - m_N}(U - 3Z)\{\}_\pi \end{aligned}$$

$$\begin{aligned}
& + \frac{Q_{\eta NN} Q_{\eta NN} A_{\Lambda N}}{m_{\Lambda} - m_N} \{\}_{\eta} \\
& + \frac{Q_{\eta \Lambda \Lambda} Q_{\eta NN} A_{\Lambda N}}{m_N - m_{\Lambda}} \{\}_{\eta}
\end{aligned} \tag{55}$$

Also we must consider vector exchange. From Figure 8 we have

$$\begin{aligned}
V_{\rho}^{(+)}(r) &= \frac{Q_{\rho \Sigma \Lambda} Q_{\rho NN} A_{\Sigma N}}{m_N - m_{\Sigma}} (U - 3Z) F_{\rho}(V, Y) \\
\text{with} \\
F_{\rho}(V, Y) &= f_{\rho}(r) + f_{\rho}(r) \frac{1}{2} \left(\frac{i\vec{\nabla}_1 - i\vec{\nabla}_2}{2M} \right)^2 + \frac{1}{2} \vec{f}_{\rho}^{(-)}(r) \cdot \frac{i\vec{\nabla}_1 - i\vec{\nabla}_2}{2M} \\
&- \frac{1}{4} (1 + \chi_Y + \chi_V) f_{\rho}^{[jj]}(r) - \frac{1}{6} (1 + \chi_Y) (1 + \chi_V) f_{\rho}^{[jj]}(r) (T - 3S) \\
&+ \frac{1}{4} (1 + \chi_Y) (1 + \chi_V) (\sigma_{\Lambda}^j \sigma_N^k - \frac{1}{3} \vec{\sigma}_{\Lambda} \cdot \vec{\sigma}_N \delta^{jk}) f_{\rho}^{[jk]}(r) T \\
&+ \frac{i}{2} [(1 + \frac{3}{2} \chi_Y) \sigma_{\Lambda}^n + (1 + \frac{3}{2} \chi_V) \sigma_N^n] \epsilon_{\ell mn} f_{\rho}^{[\ell]}(r) \frac{i\vec{\nabla}_1^m - i\vec{\nabla}_2^m}{2M}
\end{aligned} \tag{56}$$

where

$$f_M^{[j]}(r) = \left[\frac{i\vec{\nabla}_1^j - i\vec{\nabla}_2^j}{2M}, f_M(r) \right] \tag{57}$$

Again to this must be added the various ω, K^* exchange terms. Then

$$\begin{aligned}
V_{V-tot}^{(+)}(r) &= V_{\rho}^{(+)}(r) + \frac{Q_{\rho NN} Q_{\rho NN} A_{\Lambda N}}{m_{\Lambda} - m_N} (U - 3Z) F_{\rho}(V, V) \\
&+ \frac{Q_{K^* N \Lambda} Q_{K^* \Sigma N} A_{\Sigma N}}{m_N - m_{\Sigma}} (U - Z) F_{K^*}(T, G) \\
&+ \frac{Q_{K^* N \Lambda} Q_{K^* N \Lambda} A_{\Lambda N}}{m_N - m_{\Lambda}} (U - Z) F_{K^*}(T, T) \\
&+ \frac{Q_{\omega NN} Q_{\omega NN} A_{\Lambda N}}{m_{\Lambda} - m_N} F_{\omega}(Z, Z) \\
&+ \frac{Q_{\omega \Lambda \Lambda} Q_{\omega NN} A_{\Lambda N}}{m_N - m_{\Lambda}} F_{\omega}(L, Z)
\end{aligned} \tag{58}$$

We must also consider the various double pole diagrams. For the $K\pi$ term we have

$$V_{K\pi}(r) = -\frac{1}{4} \frac{Q_{\pi NN} Q_{K N \Lambda} A_{\pi K}}{m_K^2 - m_{\pi}^2} (U - 3Z) \sigma_{\Lambda}^j \sigma_N^k$$

Figure 8: ρ -exchange contribution to parity-conserving $\Lambda N \rightarrow NN$.

$$\times \left[\frac{i\nabla_1^k - i\nabla_2^k}{2M}, \left[\frac{i\nabla_1^j - i\nabla_2^j}{2M}, f_K(r) - f_\pi(r) \right] \right] \quad (59)$$

to which must be added the $V_{K\eta}$ pieces

$$\begin{aligned} V_{K\eta}(r) &= -\frac{1}{4} \frac{Q_{K\Lambda\Lambda} Q_{\eta NN} A_{\eta K}}{m_K^2 - m_\pi^2} \sigma_\Lambda^j \sigma_N^k \\ &\times \left[\frac{i\nabla_1^k - i\nabla_2^k}{2M}, \left[\frac{i\nabla_1^j - i\nabla_2^j}{2M}, f_K(r) - f_\eta(r) \right] \right] \end{aligned} \quad (60)$$

Finally we must append the $K^*\rho$ and $K^*\omega$ potentials:

$$\begin{aligned} V_{K^*\rho}(r) + V_{K^*\omega}(r) &= \frac{Q_{K^*\Lambda\Lambda} Q_{\rho NN} A_{\rho K^*}}{m_{K^*}^2 - m_\rho^2} (U - 3Z) [F_{K^*}(T, V) - F_\rho(T, V)] \\ &+ \frac{Q_{K^*\Lambda\Lambda} Q_{\omega NN} A_{\omega K^*}}{m_{K^*}^2 - m_\omega^2} (U - 3Z) [F_{K^*}(T, Z) - F_\omega(T, Z)]. \end{aligned} \quad (61)$$

Having thus constructed the ΛN “potentials” which give rise to hypernuclear decays, the task remaining is to evaluate their matrix elements in the nuclear medium, as will be described in the next section.

3 Hypernuclear Decay: Nuclear Matter Calculations

An ideal calculation of the decay of $5 \leq A \leq 40$ hypernuclei would proceed using shell-model wavefunctions for the specific nucleus under consideration. However, as a first step in such a program we present here a calculation in so-called “nuclear matter”. This is useful both as an approximation to the realistic decay of heavy hypernuclei and as a calibration with previous work in the field.

The calculation is performed utilizing the potentials given in Eqns. 49-61. A Fermi gas model is assumed with $N_n = N_p$ and $k_F = 270$ MeV. We include in the initial state only the relative S-wave projection of the Λ with each of the A nucleons—indeed the Λ is very weakly bound and its wavefunction is correspondingly quite diffuse. The decay rate is then given in terms of the expression for the Λ interaction taking place at rest

$$\Gamma_{NM} = \frac{1}{(2\pi)^5} \int_0^{k_F} d^3k \int d^3k_1 \int d^3k_2 \delta^4(p_\Lambda + k - k_1 - k_2) \frac{1}{4} \sum_{s_i, s_f} |\langle f|V|i \rangle|^2 \quad (62)$$

which can be reduced to

$$\Gamma_{NM} = \frac{1}{8\pi} \left(\frac{m_\Lambda + m_N}{m_\Lambda} \right)^3 \int_0^{k_F(\frac{m_\Lambda}{m_\Lambda + m_N})} p^2 dp \sum_{\beta\alpha} R(\beta \leftarrow \alpha) \quad (63)$$

where

$$R(\beta \leftarrow \alpha) = q m_N |\langle \beta|V|\alpha \rangle|^2 \quad (64)$$

with

$$q^2 = m_N(m_\Lambda - m_N) + \frac{p^2}{2\mu_{\Lambda N}} \quad (65)$$

are the basic transition rates between the various ΛN and NN configurations— α and β —and p, q are the relative ΛN , NN momenta respectively.² There is at least one subtlety which we need to emphasize at this point having to do with the generically written Yukawa functions

$$f_M(r) = \frac{e^{-m_M r}}{4\pi r} \quad (66)$$

²Here the sum over alpha and beta involves the various spin and isospin indices. Our notation below differs from that of other authors in that we subsume spin and isospin factors into the partial rates; details are given by B. Gibson[32].

which pervade the potential forms given earlier. The origin of these terms is the Fourier transform of the usual momentum space propagator

$$f_M(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{m_M^2 + \vec{q}^2 - q_0^2} \quad (67)$$

For a typical nucleon-nucleon interaction, we have

$$\begin{aligned} q_0 &= m_N + \frac{p_1^2}{2m_N} - m_N - \frac{p_2^2}{2m_N} \\ &= \frac{(p_1 + p_2)|\vec{q}|}{2m_N} < \frac{p_F}{m_N} |\vec{q}| \ll |\vec{q}| \end{aligned} \quad (68)$$

so that

$$f_M(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{m_M^2 + \vec{q}^2} = \frac{e^{-m_M r}}{4\pi r} \quad (69)$$

as given above. However, if the meson connects to a ΛN vertex then for a Λ at rest

$$q_0 \approx \frac{1}{2}(m_\Lambda - m_N) \quad (70)$$

The Fourier transform then becomes

$$\tilde{f}_M(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{m_M^2 + \vec{q}^2 - \frac{1}{4}(m_\Lambda - m_N)^2} = \frac{e^{-\tilde{m}_M r}}{4\pi r} \quad (71)$$

where

$$\tilde{m}_M = \sqrt{m_M^2 - \frac{1}{4}(m_\Lambda - m_N)^2} \quad (72)$$

The important feature here is that the effective range of the exchange potential is *increased*, since $\tilde{m}_M < m_M$. Thus we have

<i>meson</i>	\tilde{m}_M/m_M	
π	0.76	
K	0.98	
ρ	0.99	

(73)

so that the effect is important for pions and is understood to be included in the evaluation of all the potential expressions in Eqns. 49-61.

Transition	Operator
$^1S_0 \rightarrow ^1S_0(I=1)$	$\frac{1}{4}a(q^2)(1 - \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)$
$^1S_0 \rightarrow ^3P_0(I=1)$	$\frac{1}{8}b(q^2)(\vec{\sigma}_\Lambda - \vec{\sigma}_N) \cdot \hat{q}(1 - \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)$
$^3S_1 \rightarrow ^3S_1(I=0)$	$\frac{1}{4}c(q^2)(3 + \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)$
$^3S_1 \rightarrow ^3D_1(I=0)$	$\frac{3}{2\sqrt{2}}d(q^2)(\vec{\sigma}_\Lambda \cdot \hat{q}\vec{\sigma}_N \cdot \hat{q} - \frac{1}{3}\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)$
$^3S_1 \rightarrow ^1P_1(I=0)$	$\frac{\sqrt{3}}{8}e(q^2)(\vec{\sigma}_\Lambda - \vec{\sigma}_N) \cdot \hat{q}(3 + \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)$
$^3S_1 \rightarrow ^3P_1(I=1)$	$\frac{\sqrt{6}}{4}f(q^2)(\vec{\sigma}_\Lambda + \vec{\sigma}_N) \cdot \hat{q}$

Table 2: Transition operators of allowed $\Lambda N \rightarrow NN$ transitions from relative S-states. Here \vec{q} specifies the relative momentum of the outgoing nucleons while $\vec{\sigma}_\Lambda, \vec{\sigma}_N$ operate on the $\Lambda N, NN$ vertices respectively.

At this point, it is interesting to perform a preliminary calculation. Since the Λ is taken to be in a relative S-state with respect to any of the core nucleons, the initial $^{2S+1}L_J$ configuration must be either 1S_0 or 3S_1 . As the weak interaction can either change or not change the parity, there exist six possible transitions $\alpha \rightarrow \beta$

$$\begin{aligned}
^1S_0 &\rightarrow \begin{cases} ^3P_0 \\ ^1S_0 \end{cases} \\
^3S_1 &\rightarrow \begin{cases} ^3P_1 \\ ^1P_1 \\ ^3S_1 \\ ^3D_1 \end{cases}
\end{aligned} \tag{74}$$

We first perform the calculation for a simple pion-only-exchange potential, with the weak $\Lambda N\pi$ interaction given in Eqn. 2, which matches the on shell weak couplings observed experimentally. It is useful to characterize the various allowed $\Lambda N \rightarrow NN$ modes in terms of effective transition operators, as given in Table 2. We find then for the total nonmesonic hypernuclear decay rate

$$\Gamma_{NM} = \frac{3}{8\pi\mu_{\Lambda N}^3} \int_0^{\mu_{\Lambda N} k_F} dp p^2 m_{Nq} \left(|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + 3|f|^2 \right) \tag{75}$$

channel	$\Gamma_{NM}/\Gamma_{\Lambda}$
$^1S_0 \rightarrow ^1S_0$	0.381
$^1S_0 \rightarrow ^3P_0$	0.160
$^3S_0 \rightarrow ^3P_1$	0.319
$^3S_1 \rightarrow ^1P_1$	0.478
$^3S_1 \rightarrow ^3S_1$	0.381
$^3S_1 \rightarrow ^3D_1$	2.94

Table 3: Hypernuclear decay rates in nuclear matter for pion-only-exchange and no correlations.

	Adams[15]	McK-Gib[33]	Oset-Sal[34]	our calc.
$\frac{1}{\Gamma_{\Lambda}}\Gamma_{NM}(\text{no corr.})$	0.51	4.13	4.3	4.66
$\frac{1}{\Gamma_{\Lambda}}\Gamma_{NM}(\text{corr.})$	0.06	2.31	2.1	1.85

Table 4: Non-mesonic hypernuclear decay rates calculated by various groups using pion-exchange only in “nuclear matter.”

where $\mu_{\Lambda N} = \frac{m_{\Lambda}}{m_{\Lambda} + m_N}$ arises from the switch from the nuclear rest frame to the ΛN center of mass frame. The results of the calculation are given in Table 3. Note that about 20% of the transition rate comes from the parity violating sector and that 65% comes from the single branch $^3S_1 \rightarrow ^3D_1$!

A comparison with other pion-only-exchange calculations of hypernuclear decay in nuclear matter is provided in the first line of Table 4. We note that there exists general agreement— $(\Gamma_{NM}/\Gamma_{\Lambda})_{\pi} \sim 4.5$ —except for the calculation of Adams, whose result is nearly an order of magnitude smaller. This is primarily due to his incorrect use of a $\Lambda N \pi$ coupling which is a factor of three too small

$$g_w^{\text{Adams}} = 9.0 \times 10^{-8} \quad \text{vs.} \quad g_w^{\text{exp}} = 2.35 \times 10^{-7}. \quad (76)$$

When the correct value is used, Adams result becomes $(\Gamma_{NM}/\Gamma_{\Lambda}) = 3.6$ and agrees with the other calculations of the hypernuclear decay rate.

Our preliminary calculation is clearly naive in that it neglects both short

range correlations and the effects of additional meson exchanges. One indication of the importance of the former can be seen from a simple argument. Consider a scalar exchange between nucleons in relative S-states. We have then

$$\langle f(L=0)|V|i(L=0)\rangle \sim \mu^2 \int \psi_f^* \psi_i \frac{e^{-\mu r}}{4\pi r} d^3r - \int \psi_f^* \psi_i \delta^3(r) d^3r \quad (77)$$

Now drop the $\delta^3(r)$ component—indeed any short range correlation which gives $\psi_i(r=0) = 0$ or $\psi_f(r=0) = 0$ will eliminate $S \rightarrow S$ transitions essentially entirely, since for $\mu \sim m_\pi$

$$\int \psi_f^* \psi_i \delta^3(r) d^3r \sim 80\mu^2 \int \psi_f^* \psi_i \frac{e^{-\mu r}}{4\pi r} d^3r \quad (78)$$

This modification changes our result in Table 4 to

$$\frac{\Gamma_{NM}}{\Gamma_\Lambda}(\text{no } S \rightarrow S) = 2.18 \quad (79)$$

which is a considerable shift and emphasizes the need to consistently include the effects of initial state correlations.

Other authors have also been concerned about such correlation effects. Adams in his work used for the initial state correlation a hard core of $r_c = 0.4$ fm with a solution of the Bethe-Goldstone equation

$$f(r) \sim j_0(qr) - \frac{\sin(qr_c)\text{si}(\beta r)}{qr\text{si}(\beta r_c)} \quad (80)$$

employed for $r > r_c$. For the final nucleon-nucleon state he utilized a square well potential which also contained a hard core. An unusually strong tensor correlation was used. On the other hand McKellar and Gibson employed an effective form for the initial state correlation

$$f(r) = 1 - \exp(-\alpha r^2) \quad \text{with} \quad \alpha = 1.8 \text{ fm}^{-2} \quad (81)$$

and a Reid soft-core potential for the final state correlation. Likewise below we shall utilize an initial state correlation of the McKellar-Gibson form while for the final state we use a Reid soft-core potential to generate the final state interaction. Performing the calculation with either or both correlations

	no corr.	init. st. only	fin. st. only	both corr.
$^1S_0 \rightarrow ^1S_0$	0.381	0.0002	0.0007	0.0002
$^1S_0 \rightarrow ^3P_0$	0.160	0.070	0.048	0.037
$^3S_1 \rightarrow ^3P_1$	0.319	0.140	0.180	0.119
$^3S_1 \rightarrow ^1P_1$	0.478	0.209	0.167	0.130
$^3S_1 \rightarrow ^3S_1/^3D_1$	0.381	0.0002	2.02	0.797
$^3S_1 \rightarrow ^3S_1/^3D_1$	2.94	1.76	0.838	0.760
Total	4.66	2.18	3.25	1.85

Table 5: $\Gamma_{NM}/\Gamma_\Lambda$ in nuclear matter for pion-only-exchange and including the effects of correlations.

included, we find the results given in Table 5. Note the almost complete suppression of $S \rightarrow S$ transitions (with final state tensor interactions included, the effect is obscured by the mixing of 3S_1 and 3D_1 states.)

The comparison with other calculations, including correlations is given in line two of Table 4. We see again that there is general agreement— $(\Gamma_{NM}/\Gamma_\Lambda)_\pi^{\text{corr.}} \sim 2.0$ —except for that of Adams, for which the origin of the discrepancy is now twofold. The first problem is the use of an incorrect weak coupling, as mentioned earlier. The other is the use of an inappropriately strong tensor correlation, which suppresses the very important $^3S_1 \rightarrow ^3D_1$ transition. When both effects are corrected Adams’ number becomes $(\Gamma_{NM}/\Gamma_\Lambda)_\pi^{\text{corr.}} \sim 1.7$ and is in agreement with other authors.

From this initial pion-exchange-only nuclear matter calculation then we learn that the basic nonmesonic decay rate is anticipated to be of the same order as that for the free lambda *and* the important role played by correlations. A second quantity of interest which emerges from such a calculation is the p/n stimulated decay ratio, which has been calculated by two of the groups, results of which are displayed in Table 6. An interesting feature here is that the numbers come out so large—proton stimulated decay is predicted to predominate over its neutron stimulated counterpart by nearly an order of magnitude. The reason for this is easy to see.[32] In a pion-exchange-only scenario the effective weak interaction is of the form

$$\mathcal{H}_w \sim g \bar{N} \vec{\tau} N \cdot \bar{N} \vec{\tau} \Lambda \quad (82)$$

	Adams[15]	McK-Gib[33]	Oset-Sal[34]	our calc.
$\Gamma_{NM}(p/n)$ (no corr.)	19.4	-	-	11.2
$\Gamma_{NM}(p/n)$ (corr.)	2.8	-	-	16.6

Table 6: Proton to neutron stimulated decay ratios for pion-only exchange in “nuclear matter.”

Then $\Lambda n \rightarrow nn \sim g$ but $\Lambda p \rightarrow np \sim (-1 - (\sqrt{2})^2)g = -3g$ since both charged and neutral pion exchange are involved. In this naive picture then we have $\Gamma_{NM}(p/n) \sim 9$, in rough agreement with the numbers given in Table 6.

Armed finally with theoretical expectations, we can ask what does experiment say? The only reasonably precise results obtained for nuclei with $A > 4$ were those measured at BNL and summarized in Table 1. We observe that the measured nonmesonic decay rate is about a factor of two lower than that predicted in Table 4 while the p/n stimulation ratio differs by at least an order of magnitude from that given in Table 6. The problem may be, of course, associated with the difference between the nuclear matter within which the calculations were performed and the finite nuclear systems which were examined experimentally. Or it could be due to the omission of the many shorter range exchanged mesons in the theoretical estimate. (Or *both*!)

Before undertaking the difficult problem of finite nuclear calculations, it is useful to first examine the inclusion of additional exchanged mesons in our calculations. As mentioned above, a primary difficulty in this approach is that none of the required weak couplings can be measured experimentally. Thus the use of some sort of model is required, and the significance of any theoretical predictions will be no better than the validity of the model. One early attempt by McKellar and Gibson,[33] for example, included only the rho and evaluated the rho couplings using SU(6) and alternatively the straightforward but flawed factorization approach. Well aware of the limitations of the latter they allowed an arbitrary phase between the rho and pi amplitudes and renormalized the factorization calculation by a factor of $1/\sin\theta_c \cos\theta_c$ in order to account for the $\Delta I = \frac{1}{2}$ enhancement. Obviously this is only a rough estimate then and this is only for the rho meson exchange contribution! A similar approach was attempted by Nardulli, who calculated the parity-conserving rho amplitude in a simple pole model and the parity

	McK-Gib[33] $\pi + \rho$	McK-Gib[33] $\pi - \rho$	Nard.[32]
$\frac{1}{\Gamma_\Lambda} \Gamma_{NM}$	3.52	0.72	0.7

Table 7: Nonmesonic decay rates in nuclear matter in pi plus rho exchange models

Exchanged Meson	S→P	S→S/D	Total
π	0.288	1.56	1.85
$+\eta$	0.320	1.36	1.68
$+K$	0.568	0.74	1.31
$+\rho$	0.523	0.59	1.11
$+\omega$	0.576	0.61	1.19
$+K^*$	0.721	0.66	1.38

Table 8: Hypernuclear decay rates— $\Gamma_{NM}/\Gamma_\Lambda$ —in nuclear matter including the effects of correlations and with the contributions of non-pion exchanges.

violating piece in a simple quark picture.[35] Results of these calculations are shown in Table 7.

To our knowledge, the only comprehensive calculation which has been undertaken to date is that of our group, which is described in the previous section of this paper. We show in Table 8 the results of including the effects of additional meson exchange contributions in arbitrary order. We see from Table 8 that inclusion of *all* pseudoscalar and vector meson exchanges in addition to the long-range pion-exchange component has the effect of reducing the hypernuclear decay rate to a value about 40% above that for free Λ -decay and in agreement with the $A \sim 12$ results. However, in view of the theoretical uncertainties associated with our calculation and the fact that it is performed in nuclear matter, this agreement cannot be said to distinguish between the pion-only-exchange and all-exchanges scenarios.

A more convincing case for the presence of non-pion-exchange *can* be constructed by examining the proton/neutron stimulation ratio. On the the-

	$\Gamma_{NM}(p/n)$	$\Gamma_{NM}(PV/PC)$
π (no corr.)	11.2	0.14
π (with corr.)	16.6	0.18
$\pi + \rho$	13.1	0.21
$\pi, \rho, \omega, \eta, K, K^*$	2.9	0.90

Table 9: The parity violating to parity conserving and p to n ratios for hypernuclear decay in “nuclear matter.”

oretical side, we find

$$\Gamma_{NM}(p/n) = \frac{\int_0^{\mu_{\Lambda N} k_F} p^2 dp q (|a|^2 + |b|^2 + 3|c|^2 + 3|d|^2 + 3|e|^2 + 3|f|^2)}{\int_0^{\mu_{\Lambda} k_F} p^2 dp q (2|a|^2 + 2|b|^2 + 6|f|^2)} \quad (83)$$

in terms of the couplings $a, b, \dots f$ defined in Table 2. Results are shown in Table 9. We observe that inclusion of additional exchanges plays a major role in reducing the p/n ratio from its pion-only-exchange value. The resulting value of 2.9 is still somewhat larger than the experimental numbers shown in Table 1 but clearly indicates the presence of non-pion-exchange components.

The reason that kaon exchange in particular can play such a major role can be seen from a simple argument due to Gibson[32] who pointed out that since the final NN system can have either $I=0$ or $I=1$, the effective kaon exchange interaction can be written as

$$\begin{aligned} \mathcal{L}_{eff} &= A_0(\bar{p}p + \bar{n}n)\bar{n}\Lambda + A_1(2\bar{n}p\bar{p}\Lambda - (\bar{p}p - \bar{n}n)\bar{n}\Lambda) \\ &\sim (A_0 - 3A_1)\bar{p}p\bar{n}\Lambda + (A_0 + A_1)\bar{n}n\bar{n}\Lambda \end{aligned} \quad (84)$$

where the second line is obtained via a Fierz transformation. Since for parity-violating kaon exchange we have $A_0 \sim 6.8A_1$ ³ we find[32]

$$\Gamma_{NM}(p/n) = \left(\frac{A_0 - 3A_1}{A_0 + A_1} \right)^2 \sim 1/4 \quad (86)$$

³From Eqs. 34,40 we determine

$$\frac{A_0}{A_1} = \frac{C_{KNN} + 2D_{KNN}}{C_{KNN}} = 3\left(1 - \frac{b_V}{c_V}\right) \simeq 6.8 \quad (85)$$

	π -no corr.	π -corr.	all exch.
α	-0.078	-0.192	-0.443

Table 10: Proton asymmetry coefficient in various scenarios.

which clearly indicates the importance of inclusion of non-pion-exchange components in predicting the p/n ratio.

A second strong indication of the presence of non-pion-exchange can be seen from Table 9 in that the ratio of rates for parity-violating to parity-conserving transitions is substantially enhanced by the inclusion of kaon and vector meson exchange as compared to the simple pion-exchange-only calculation. We can further quantify this effect by calculating explicitly the angular distribution of the emitted proton in the $\Lambda p \rightarrow np$ transition (there can be no asymmetry for the corresponding $\Lambda n \rightarrow nn$ case due to the identity of the final state neutrons), yielding

$$W_p(\theta) \sim 1 + \alpha P_\Lambda \cos \theta \quad (87)$$

where

$$\alpha = \frac{\int_0^{\mu_{\Lambda N} k_F} p^2 dp q \frac{\sqrt{3}}{2} \text{Re} f^*(\sqrt{2}c + d)}{\int_0^{\mu_{\Lambda N} k_F} p^2 dp q \frac{1}{4} (|a|^2 + |b|^2 + 3|c|^2 + 3|d|^2 + 3|e|^2 + 3|f|^2)} \quad (88)$$

is the asymmetry parameter. Results of a numerical evaluation are shown in Table 10 so that again inclusion of non-pion-exchange components has a significant effect, increasing the expected $\Lambda p \rightarrow np$ asymmetry by more than a factor of two. This prediction of a substantial asymmetry is consistent with preliminary results obtained for p-shell nuclei at KEK.[36]

4 Hypernuclear Decay: Additional Considerations

Although nuclear matter calculations are of great interest in identifying basic properties of the decay process, true confrontation with experiment requires

	Oset-Sal[34]	TRIUMF[37]	our calc.
$\frac{1}{\Gamma_\Lambda}\Gamma_{NM}\pi$ (no corr.)		1.6	3.4
π (corr.)	1.5	2.0(1.0)	0.5
$\pi + K$ [37]; $\pi, \eta, \rho\omega, K, K^*$ [7]		1.2	0.2
$\Gamma_{NM}(p/n)\pi$ (no corr.)		5.0	4.6
π (corr.)		5.0	5.0
$\pi + K$ [37]; $\pi, \eta, \rho, \omega, K, K^*$ [7]		4.0	1.2
$\Gamma_{NM}(PV/PC)\pi$ (no corr.)		0.4	0.1
π (corr.)		0.5	0.1
$\pi + K$ [17]; $\pi, \eta, \rho, \omega, K, K^*$ [7]		0.3	0.8

Table 11: Calculated properties of nonmesonic hypernuclear decay of $^{12}_\Lambda\text{C}$.

calculations involving the finite nuclei on which the measurements are conducted. Of course, such calculations are considerably more demanding than their nuclear matter counterparts and require Λ shell model considerations as well as non-S-shell capture. Nevertheless a number of groups have taken up the challenge. Details of our own calculation in $^{12}_\Lambda\text{C}$ and $^5_\Lambda\text{He}$ will be presented in a future publication. These are performed using a simple shell model to describe the hypernuclear structure, where only an extreme single particle model with no configuration mixing and only phenomenological forms for the correlation functions are employed. In Table 11 we compare our preliminary results for $^{12}_\Lambda\text{C}$ with that obtained in a parallel calculation performed by a TRIUMF collaboration[37] and with a pion-only-exchange version by Oset and Salcedo.[34] In comparing with the experimental results given in Table 3, we see that our calculation is certainly satisfactory, but the discrepancy with the TRIUMF work is disturbing and needs to be rectified before either is to be believed.

A second nucleus on which there has been a good deal of work, both experimentally and theoretically, is $^5_\Lambda\text{He}$, which is summarized in Table 12. Here again what is important is not so much the agreement or disagreement with experiment but rather the discrepancies *between* the various calculations which need to be clarified before any significant confrontation between theory and experiment is possible. One step in this regard has already been

	Oset-Sal[34]	TRIUMF[37]	TTB[38]	our calc.
$\frac{1}{\Gamma_\Lambda}\Gamma_{NM}\pi(\text{no corr.})$		1.0	0.5	1.6
$\pi(\text{corr.})$	1.15	0.25(0.5)	0.144	0.9
$\pi + K[37]; \pi, \eta, \rho, \omega, K, K^*[7]$		0.22		0.5
$\Gamma_{NM}(p/n)\pi(\text{no corr.})$		5.0		15
$\pi(\text{corr.})$		4.8		19
$\pi + K[37]; \pi, \eta, \rho, \omega, K, K^*[7]$		5.4		2.1

Table 12: Calculated properties of the nonmesonic decay of ${}^5_\Lambda\text{He}$.

taken in that the TRIUMF collaboration have reexamined their work and have discovered an inconsistency between their calculational technique and the correlations which were utilized.[39] When the correlations are properly included the substantial reductions in the calculated finite hypernuclear decay rates given in Table 12 are replaced by values which are much more in accord with our calculation, as shown in parentheses. Only the pion exchange piece has been included and there is clearly much room for additional work.

Before leaving this section, however, it is important to raise an additional issue which must be resolved before reliable theoretical calculations are possible—that of the $\Delta I = \frac{1}{2}$ rule.[40] Certainly in any venue in which it has been tested—nonleptonic kaon decay— $K \rightarrow 2\pi, 3\pi$, hyperon decay— $B \rightarrow B'\pi$, $\Delta I = \frac{1}{2}$ components of the decay amplitude are found to be a factor of twenty or so larger than their $\Delta I = \frac{3}{2}$ counterparts. Thus it has been natural in theoretical analysis of nonmesonic hypernuclear decay to make this same assumption. (Indeed without it the already large number of unknown parameters in the weak vertices expands by a factor of two.) However, recently Schumacher has raised a serious question about the correctness of this assumption, which if verified will have serious implications about the direction of future theoretical analyses.[41] The point is that by use of very light hypernuclear systems one can isolate the isospin structure of the weak transition. Specifically, using a simple delta function interaction model of the hypernuclear weak decay process, as first written down by Block and Dalitz

in 1963, one determines[42]

$$\begin{aligned}
{}^4_{\Lambda}\text{He} : \gamma_4 &= \Gamma_{NM}(n/p) = \frac{2R_{n0}}{3R_{p1} + R_{p0}} \\
{}^5_{\Lambda}\text{He} : \gamma_5 &= \Gamma_{NM}(n/p) = \frac{3R_{n1} + R_{n0}}{3R_{p1} + R_{p0}} \\
\gamma &= \Gamma_{NM}({}^4_{\Lambda}\text{He})/\Gamma_{NM}({}^4_{\Lambda}\text{H}) = \frac{3R_{p1} + R_{p0} + 2R_{n0}}{3R_{n1} + R_{n0} + 2R_{p0}}
\end{aligned} \tag{89}$$

where here R_{Nj} indicates the rate for N-stimulated hypernuclear decay from an initial configuration having spin j. One can then isolate the ratio R_{n0}/R_{p0} by taking the algebraic combination

$$\frac{R_{n0}}{R_{p0}} = \frac{\gamma\gamma_4}{1 + \gamma_4 - \gamma\gamma_5} \tag{90}$$

and from the experimental values[43]

$$\gamma_4 = 0.27 \pm 0.14, \quad \gamma_5 = 0.93 \pm 0.55, \quad \gamma = 0.73^{+0.71}_{-0.22} \tag{91}$$

one determines

$$\frac{R_{n0}}{R_{p0}} = \frac{0.20^{+0.22}_{-0.12}}{0.59^{+0.80}_{-0.47}} \tag{92}$$

in possible conflict with the $\Delta I = \frac{1}{2}$ rule prediction— $R_{n0}/R_{p0} = 2$.⁴ If confirmed by further theoretical and experimental analysis this would obviously have important ramifications for hypernuclear predictions. However, recent work at KEK has indicated that the correct value for γ should be near to unity rather than the value 0.5 used above in which case the ratio is considerably increased and there may no longer be any indication of $\Delta I = \frac{1}{2}$ rule violation.[44]

5 Conclusions

The subject of hypernuclear weak decay has recently undergone somewhat of a renaissance. During the early 1960's there existed a healthy combination

⁴Note that final state nn or np configurations which arise from initial 1S_0 states are of necessity I=1.

of emulsion based experiments and theoretical work by Dalitz' Oxford group and by Adams at Stanford. Then for nearly two decades there was a dearth of activity in this area. The situation changed during the 1980's, with the fast counting experiments mounted at Brookhaven by the CMU-BNL-UNM-Houston-Texas-Vassar collaboration, which renewed interest in this subject. The work has generated a good deal of theoretical activity, as described above, as well as a variety of new and proposed experiments.

Above we described our own calculational program, which utilizes a meson exchange picture of the $\Lambda N \rightarrow NN$ process in hypernuclei. Because we are dealing with a $\Delta S = 1$ weak transition, there exist a variety of mesons which must be considered, both strange and non-strange. The weak BB'M vertices were evaluated using the same $SU(6)_w$ and quark model techniques which have been utilized with some success in the treatment of the somewhat similar problem of nuclear parity violation. In comparing the predictions of our calculations and those of other groups with the limited experimental data, it must be acknowledged that the present situation is unsatisfactory. Although there is very rough qualitative agreement between theoretical expectations and experimental measurements, it is not clear whether those discrepancies which do exist are due to experimental uncertainties, to theoretical insufficiencies, or both. On the theoretical side, what is needed are reliable calculations on finite hypernuclei (preferably by more than one group) which clearly indicate the signals that should be sought in the data. The issue associated with the validity of the $\Delta I = \frac{1}{2}$ rule must be clarified. In addition there have been recent speculations about the possible significance of two-nucleon stimulated decay[45] (which could account for as much as 15% of the decay amplitude according to estimates) and of the importance of direct quark (non-meson-exchange) mechanisms,[46] which deserve further study in order to eliminate the vexing double counting problems which arise when both are included. Proper treatment of the former involves a careful treatment of the many-body aspects of the nuclear medium in the presence of the weak interaction and will require study by nuclear theorists. On the other hand, the sorting out of direct quark vs. meson-exchange components of the weak amplitude is an extremely subtle problem requiring the efforts of both particle and nuclear theorists. For example, recent works by Maltman and Shmatikov[30] and by Inoue, Takeuchi and Ota[31] calculate so-called "direct" four-quark contributions to weak hypernuclear decay by evaluating the (renormalization group corrected) local Hamiltonian in the context of a sim-

Figure 9: Quark picture of meson exchange contribution to nonmesonic hyperon decay.

ple quark model for the baryon states. The problems with such an approach are at least two. Firstly, as discussed earlier in this paper, this technique cannot account for the validity of the $\Delta I = \frac{1}{2}$ rule in other contexts—without *soft* gluonic effects properly included one cannot expect this simple picture to generate a realistic picture of hypernuclear decay. The second problem is that the "direct" mechanism omits the important contributions associated with meson exchange, as calculated in our work. In a quark picture these are associated with diagrams such as that shown in Figure 9. In order to avoid double counting issue, such pieces must be carefully subtracted from any "direct" quark calculation before combining "direct" and meson exchange components. This is a highly non-trivial assignment which has yet to be solved, and which we relegate to future work.

On the experimental side, we require an extensive and reliable data base developed in a variety of nuclei in order to confirm or refute the predicted patterns. The existence of new high intensity accelerator facilities such as

CEBAF and DAΦNE will be important in this regard. In any case it is clear that hypernuclear weak decay is a field in its infancy and that any present theoretical optimism may well all too soon be tempered by experimental reality.

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